# Notation List 

## For Cambridge International Mathematics Qualifications

For use from 2020

## Mathematical notation

Examinations for CIE syllabuses may use relevant notation from the following list.

## 1 Set notation

```
\epsilon
is an element of
\(\notin \quad\) is not an element of
\(\left\{x_{1}, x_{2}, \ldots\right\} \quad\) the set with elements \(x_{1}, x_{2}, \ldots\)
\(\{x: \ldots\} \quad\) the set of all \(x\) such that \(\ldots\)
\(\mathrm{n}(A)\)
\(\varnothing\)
\(\mathscr{E}\)
U
\(A^{\prime}\)
\(\mathbb{N}\)
\(\mathbb{Z}\)
Q
R
\(\mathbb{C}\)
\((x, y)\)
\(\subseteq\)
c
U
ก
\([a, b] \quad\) the closed interval \(\{x \in \mathbb{R}: a \leqslant x \leqslant b\}\)
\([a, b) \quad\) the interval \(\{x \in \mathbb{R}: a \leqslant x<b\}\)
\((a, b] \quad\) the interval \(\{x \in \mathbb{R}: a<x \leqslant b\}\)
\((a, b) \quad\) the open interval \(\{x \in \mathbb{R}: a<x<b\}\)
\((S, \circ \quad \quad\) or group consisting of the set \(S\) with binary operation 。
```


## 2 Miscellaneous symbols

```
= is equal to
# is not equal to
\equiv is identical to or is congruent to
\approx is approximately equal to
~ is distributed as
\cong is isomorphic to
\propto is proportional to
< is less than
\leqslant}\quad\mathrm{ is less than or equal to
> is greater than
\geqslant is greater than or equal to
\infty}\quad\mathrm{ infinity
m implies
\Leftarrow\quad is implied by
\Leftrightarrow implies and is implied by (is equivalent to)
```


## 3 Operations

$a+b$
$a-b$
$a \times b, a b$
$a \div b, \frac{a}{b}$
$\sum_{i=1}^{n} a_{i}$
$\sqrt{a} \quad$ the non-negative square root of $a$, for $a \in \mathbb{R}, a \geqslant 0$
$\sqrt[n]{a} \quad$ the (real) $n$th root of $a$, for $a \in \mathbb{R}$, where $\sqrt[n]{a} \geqslant 0$ for $a \geqslant 0$
$|a| \quad$ the modulus of $a$
$n!\quad n$ factorial
$\binom{n}{r}$
$a$ plus $b$
$a$ minus $b$
$a$ multiplied by $b$
$a$ divided by $b$
$a_{1}+a_{2}+\ldots+a_{n}$
the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}$ and $0 \leqslant r \leqslant n$

## 4 Functions

$\mathrm{f}(x)$
$\mathrm{f}: A \rightarrow B$
$\mathrm{f}: x \mapsto y$
$\mathrm{f}^{-1}$
gf
$\lim _{x \rightarrow a} \mathrm{f}(x)$
$\Delta x, \delta x$
$\frac{\mathrm{d} y}{\mathrm{~d} x}$
$\frac{\mathrm{d}^{n} y}{\mathrm{~d} x^{n}}$
$\mathrm{f}^{\prime}(x), \mathrm{f}^{\prime \prime}(x), \ldots, \mathrm{f}^{(n)}(x)$
$\int y \mathrm{~d} x \quad$ the indefinite integral of $y$ with respect to $x$
$\int_{a}^{b} y \mathrm{~d} x$
$\dot{x}, \ddot{x}, \ldots$
the value of the function f at $x$
the function f maps the element $x$ to the element $y$
the inverse function of the one-one function $f$
the composite function of f and g which is defined by $\operatorname{gf}(x)=\mathrm{g}(\mathrm{f}(x))$
the limit of $\mathrm{f}(x)$ as $x$ tends to $a$
an increment of $x$
the derivative of $y$ with respect to $x$
the $n$th derivative of $y$ with respect to $x$
the first, second, ..., $n$th derivatives of $\mathrm{f}(x)$ with respect to $x$
the first, second, ... derivatives of $x$ with respect to $t$
f is a function under which each element of set $A$ has an image in set $B$
the definite integral of $y$ with respect to $x$ between the limits $x=a$ and $x=b$

## 5 Exponential and logarithmic functions

e
$\mathrm{e}^{x}, \exp (x)$
$\log _{a} x$
$\ln x$
$\lg x, \log _{10} x$
base of natural logarithms
exponential function of $x$
logarithm to the base $a$ of $x$
natural logarithm of $x$
logarithm of $x$ to base 10

## 6 Circular and hyperbolic functions

| $\left.\begin{array}{l}\sin , \cos , \tan \\ \begin{array}{l}\operatorname{cosec}, \sec , \cot t\end{array} \\ \sin ^{-1}, \cos ^{-1}, \tan ^{-1} \\ \operatorname{cosec}^{-1}, \sec ^{-1}, \cot ^{-1}\end{array}\right\} \quad$ the circular functions |
| :--- |
| $\left.\begin{array}{l}\text { sinh, cosh, tanh } \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth}\end{array}\right\}$ |
| $\left.\begin{array}{l}\sinh ^{-1}, \cosh ^{-1}, \tanh ^{-1} \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \operatorname{coth}^{-1}\end{array}\right\}$ the inverse circular functions | the inverse hyperbolic functions

## 7 Complex numbers

i
$z$
$\operatorname{Re} z$
$\operatorname{Im} z$
$|z|$
$\arg z$
$z^{*}$

## 8 Matrices

## M

$\mathbf{M}^{-1}$
$\operatorname{det} \mathbf{M},|\mathbf{M}|$
I

## 9 Vectors

| a | the vector a |
| :---: | :---: |
| $\overrightarrow{A B}$ | the vector represented in magnitude and direction by the directed line segment $A B$ |
| â | a unit vector in the direction of $\mathbf{a}$ |
| $\mathbf{i}, \mathbf{j}, \mathbf{k}$ | unit vectors in the directions of the Cartesian coordinate axes |
| $\binom{x}{y},\left(\begin{array}{l} x \\ y \\ z \end{array}\right)$ | the vectors $x \mathbf{i}+y \mathbf{j}$ (in 2 dimensions) and $x \mathbf{i}+y \mathbf{j}+z \mathbf{k}$ (in 3 dimensions) |
| $\|\mathbf{a}\|, a$ | the magnitude of $\mathbf{a}$ |
| $\|\overrightarrow{A B}\|, A B$ | the magnitude of $\overrightarrow{A B}$ |
| a.b | the scalar product of $\mathbf{a}$ and $\mathbf{b}$ |
| $\mathbf{a} \times \mathbf{b}$ | the vector product of $\mathbf{a}$ and $\mathbf{b}$ |

## 10 Probability and statistics

| $A, B, C, \ldots$ | events |
| :---: | :---: |
| $A \cup B$ | union of the events $A$ and $B$ |
| $A \cap B$ | intersection of the events $A$ and $B$ |
| $\mathrm{P}(A)$ | probability of the event $A$ |
| $A^{\prime}$ | complement of the event $A$ |
| $\mathrm{P}(A \mid B)$ | probability of the event $A$ conditional on the event $B$ |
| ${ }^{n} \mathrm{C}_{r}$ | the number of combinations of $r$ objects from $n,{ }^{n} \mathrm{C}_{r}=\binom{n}{r}=\frac{n!}{r!(n-r)!}$ |
| ${ }^{n} \mathrm{P}_{r}$ | the number of permutations of $r$ objects from $n,{ }^{n} \mathrm{P}_{r}=\frac{n!}{(n-r)!}$ |
| $X, Y, R, \ldots$ | random variables |
| $x, y, r, \ldots$ | values of the random variables $X, Y, R, \ldots$ |
| $x_{1}, x_{2}, \ldots$ | observations |
| $f_{1}, f_{2}, \ldots$ | frequencies with which the observations $x_{1}, x_{2}, \ldots$ occur |
| $\mathrm{p}(x)$ | probability function $\mathrm{P}(X=x)$ of the discrete random variable $X$ |
| $p_{1}, p_{2}, \ldots$ | probabilities of the values $x_{1}, x_{2}, \ldots$ of the discrete random variable $X$ |
| $\mathrm{f}(x)$ | value of the probability density function of a continuous random variable $X$ |
| $\mathrm{F}(x)$ | value of the cumulative distribution function of a continuous random variable $X$ |
| $\mathrm{E}(X)$ | expectation of the random variable $X$ |
| $\mathrm{E}(\mathrm{g}(X))$ | expectation of $\mathrm{g}(X)$ |
| $\operatorname{Var}(X)$ | variance of the random variable $X$ |
| $\mathrm{G}_{X}(t)$ | probability generating function for the discrete random variable $X$ |
| $\mathrm{M}_{X}(t)$ | moment generating function for the random variable $X$ |
| $\mathrm{B}(n, p)$ | binomial distribution with parameters $n$ and $p$ |
| Geo(p) | geometric distribution with parameter $p$ |
| $\mathrm{Po}(\lambda)$ | Poisson distribution with parameter $\lambda$ |
| $\mathrm{N}\left(\mu, \sigma^{2}\right)$ | normal distribution with mean $\mu$ and variance $\sigma^{2}$ |
| $\mu$ | population mean |
| $\sigma^{2}$ | population variance |
| $\sigma$ | population standard deviation |
| $\bar{x}$ | sample mean, $\bar{x}=\frac{1}{n} \sum_{i=1}^{n} x_{i}$ |
| $s^{2}$ | unbiased estimate of population variance from a sample, $s^{2}=\frac{1}{n-1} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$ |
| $\rho$ | product moment correlation coefficient for a population |
| $r$ | product moment correlation coefficient for a sample |
| $\phi$ | probability density function of the standardised normal variable $Z \sim \mathrm{~N}(0,1)$ |
| $\Phi$ | cumulative distribution function of the standardised normal variable $Z \sim \mathrm{~N}(0,1)$ |
| $\mathrm{H}_{0}, \mathrm{H}_{1}$ | null and alternative hypotheses for a hypothesis test |

